Work and time sample analogy:

Let the time taken by A to finish a work be x days.

Let the time taken by B to finish the same work be y days.

Work done by A in 1 day $=(1/x)$

Work done by B in 1 day $=(1/y)$

Hence workdone by A and B together in 1 day $=(1/x)+(1/y)=(x+y)/xy$

For completing the fraction of a work ,that is $(x+y)/xy$,time taken by A and B =1 day

Hence for completing the whole work,time taken by A and B = $1/((x+y)/xy)= xy/(x+y)$……………(1)

When working alone, If A takes 'a' days more than the time taken by A and B together,

$x = xy/(x+y) + a$ (Using equation (1))

$x(x+y) = xy +a(x+y)$

$x^2 = a(x+y)$

$x = \sqrt{(a(x+y))}$…………………………………….(2)

$a= x^2/(x+y)$ ……………………………………(3)

When working alone, If B takes 'b' days more than the time taken by A and B together,

$y = xy/(x+y) + b$ (Using equation (2))

$y(x+y) = xy +b(x+y)$

$y^2 = b(x+y)$

$y= \sqrt{(b(x+y))}$………………………………………….(4)

$b = y^2/(x+y)$……………………………………………(5)

From equations (2) and (4),

We get,

$xy/(x+y) = \sqrt{(a(x+y))}*\sqrt{(b(x+y))}/ (\sqrt{(a(x+y)}+\sqrt{(b(x+y))})$

$= \sqrt{a}*\sqrt{b}*\sqrt{(x+y)}/(\sqrt{a}+\sqrt{b})$

$= \sqrt{a}*\sqrt{b}*\sqrt{(x+y)}/((x/\sqrt{(x+y)})+(y/\sqrt{(x+y)}))$

$=\sqrt{a}*\sqrt{b}$………………………………………………….(6)

Example : If A takes 10 days and B takes 15 days to complete a work,

  a) How many more days does A take than the days taken by A and B when working together?
  b) How many more days does B take than the days taken by A and B when working together?

Solution:

a) Number of days taken by A more than the days taken by A and B together= $10^2/(10+15)$
   = (100/25)=4 (From equation (3))
b) Number of days taken by B more than the days taken by A and B together = $15^2/(10+15)$
c) =(225/25)=9 (From equation (5))

Example: If A and B takes 4 and 16days respectively more than the days taken by A and B working

together,

a) What is the time taken by A and B together?
b) What is the time taken by A and B alone respectively?

Solution:

a) Time taken by A and B together to complete the job= $\sqrt{64}$ = 8 days (From equation (6)).
b) Time taken by A alone = (8+4)=12 days.
   Time taken by B alone= (8+16)= 24 days.